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Observer-Based Adaptive Fault-Tolerant Control of A Class of Nonlinear Systems With Actuator Failures

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***Abstract*—The output feedback tracking control problem is studied for a class of uncertain nonlinear systems with actuator failures. An adaptive observer is designed to reconstruct immeasureable state information of the system, and an observer-based adaptive fault-tolerant control (FTC) strategy is developed recursively by backstepping methods, neural networks (NNs), FTC theory and the dynamic surface control (DSC) technique. The proposed strategy is only dependent on output information, and there is no requirement for accurate parameters of the system. In theory, the stability of the closed-loop system is proven that all signals are uniformly ultimately bounded and the control scheme can force the tracking error converge to a small neighborhood of the origin.**

I. Introduction

Over the past two decades, adaptive control approaches have been widely used to deal with system faults in various systems, and one of the typical schemes is the adaptive backstopping technique [1]–[6]. The main limitation of the papers [1]–[6] is that it is assumed that actuators of the systems are well working and the faults are not taken into account. It is well known that the phenomenon of component faults frequently occur in practical industrials [7]–[9], and it may be one of major sources of instability of systems and has received considerable attention from control communications. In [10] and [11], adaptive fault-tolerant control (FTC) methods were designed for linear systems with faults of lock-in-place and loss of effectiveness. In [12] [13] and [14], Tao *et al.* and Jiang *et al.* developed adaptive fault-tolerant controllers for a class of nonlinear single-input and single-output (SISO) systems and multi-input and multi-output (MIMO) systems. However, the mentioned references [10]–[14] are effective under the condition that it is assumed that discussed systems are with matching conditions or with known nonlinear functions. To relax these assumptions, Li *et al.* reported adaptive fuzzy control ways with the help of fuzzy logical tools in [15][16]. Moreover, adaptive fuzzy output feedback FTC schemes were emerged in for several classes of uncertain nonlinear systems suffered from actuator failures [7][17][18]. Recently, a novel adaptive fault tolerant controller design was proposed for a class of nonlinear time-delay systems in [8], unknown systems with multiple actuators in [19], multi-agent systems in [20], and Markovian jump systems in [21]. However, the control methods in [7][8][15][19] are based on traditional backstepping ways, and computational burdens increase drastically along with the growth of the order of the system because of the repeat differentiation calculation of virtual functions. Fortunately, a dynamic surface control (DSC) technique was proposed by Swaroop *et al.* in [22]. The main idea of this approach was introduced via a first-order filter at each step and it was extended to control of a class of strict-feedback nonlinear systems combing with NNs in [23]. On the basis of this idea, some efforts have been made to apply this technique for more general systems, such as strict-feedback systems [25], non-affine pure-feedback systems [26][27], low-triangular MIMO systems with time delays [6][28].

Inspired by the aforementioned literature, we focus our attention to the FTC problem via a state observer. With the help of appropriate transformation, virtual control variables of the large-scale pure-feedback system are converted into dominant forms. A state observer is constructed by universal approximation theory of neural networks (NNs), and the DSC technique is also introduced to overcome the so-called ‘explosion of complexity’ utilizing a first-order low-pass filter. Then, an adaptive NNs fault-tolerant output control strategy will be designed.

II. Problem Formulation

Consider a class of nonlinear systems suffered from actuator failures

$$\begin{cases} \dot{x}_1(t) = f_1\left(x_1(t), x_2(t)\right), \\ \dot{x}_2(t) = f_2\left(\underline{x}_2(t), x_3(t)\right), \\ \qquad\vdots \\ \dot{x}_{n-1}(t) = f_{n-1}\left(\underline{x}_{n-1}(t), x_n(t)\right), \\ \dot{x}_n(t) = f_n\left(\underline{x}_n(t)\right) + \varpi^{\mathrm{T}} \underline{u}(t), \\ y(t) = x_1(t), \end{cases} \tag{1}$$

where $x_i(t)$ are state variables of the system, $i = 1, \cdots, n$, $\underline{x}_i(t) = \left[x_1(t), \ldots, x_i(t)\right]^{\mathrm{T}} \in \mathbb{R}^i$, $f_i(\cdot) \in \mathbb{R}$ are unknown and smooth nonlinear functions denoting uncertain dynamics of the system, $\underline{u}(t) = \left[u_1(t), \cdots, u_m(t)\right]^{\mathrm{T}} \in \mathbb{R}^m$ is the control input vector whose components might be of failure during the whole process, and $y(t) \in \mathbb{R}$ is the output signal of the system. It is assumed that only the output information are available for measurement.

Denoting a set $\mathcal{A}^m = \{1, 2, \ldots, m\}$ and the actuator faults discussed in this paper include two categories: one is lock-in-place, the other one is loss of effectiveness.

Lock-in-place model:

$$u_j(t) = \bar{u}_j, t \geq t_j, j \in \{j_1, \ldots, j_q\} \subset \mathcal{A}^m, \tag{2}$$

and loss of effectiveness model:

$$u_r(t) = \kappa_r \nu_r(t), t \geq t_r, r \in \overline{\{j_1, \ldots, j_q\}} \bigcap \mathcal{A}^m, \tag{3}$$

where $\kappa_r \in [\underline{\kappa}_r, 1]$, $\bar{u}_j$ a constant value when the actuator gets stuck, $\nu_r(t)$ is the applied control variable, κ_r is the still effective proportion of the rth actuator when the loss of effectiveness fault takes place, and $\underline{\kappa}_r$ is the lower boundedness of κ_r, $0 < \underline{\kappa}_r \leq 1$.

Then the input vector becomes

$$u(t) = \kappa\nu(t) + \mu\big(\bar{u} - \kappa\nu(t)\big), \tag{4}$$

where $\nu(t) = \left[\nu_1(t), \cdots, \nu_m(t)\right]^{\mathrm{T}}$, $\bar{u} = \left[\bar{u}_1, \cdots, \bar{u}_m\right]^{\mathrm{T}}$, $\kappa = \mathrm{diag}(\kappa_1, \cdots, \kappa_m)$, $\mu = \mathrm{diag}(\mu_1, \cdots, \mu_m)$, and

$$\mu_j = \begin{cases} 1, \text{if the the} j\text{th actuator fails as (2),} \\ \quad \text{e.g., } u_j = \bar{u}_j, \\ 0, \text{otherwise.} \end{cases} \tag{5}$$

Remark 1: In practice, the general form of (1) is applicable to describe several physical plants, such as biochemical processes, robotic manipulators, unmanned surface vehicles and so on [29]. In this context, we are interested in the control design for the System (1) directly, which is not academically challenging but also of practical interest.

For the sake of notation simplicity, the time notation t is in the delay-free terms. To move on, the following standard assumption is introduced.

Assumption 1: The reference signal y_{d} is smooth and y_{d}, $\dot{y}_{\mathrm{d}}$, $\ddot{y}_{\mathrm{d}}$ are bounded, that is, there exists a positive constant ϱ_{i0}, such that $y_{\mathrm{d}}^2 + \dot{y}_{\mathrm{d}}^2 + \ddot{y}_{\mathrm{d}}^2 \leq \varrho_0$.

Remark 2: The energy of the command generators are limited, Assumption 1 is significantly common in exiting literature and can be found in [25].

The preliminaries about the radical basic function (RBF) neural networks and the universal approximation property are omitted for conciseness in this paper. Readers interested may refer to [30] and reference therein for more details. It is worth noting that W_i^* is the unknown ideal constant weight vector of RBF neural networks and $G_i(Z_i)$ is the Gaussian function with the input vector Z_i to be given later.

The control objective in this paper is to design an observer-based adaptive fault-tolerant control law for the System (1) in the presence of faults (2) and (3) to force output y to follow the desired trajectory y_{d} as expected and to guarantee all signals in the closed-loop system uniformly ultimately bounded.

III. Output Feedback Control Law

In this section, we will focus our attention to achieve the control objective described in Section II. According to Yang *et al.* [15], a kind of special control structures is chosen as

$$\nu_j = b_j(\underline{\hat{x}}_n)u_0, \tag{6}$$

where $0 < \underline{b}_j \leq b_j(\underline{\hat{x}}_n) \leq \bar{b}_j$, $\underline{b}_j > 0$, $\bar{b}_j > 0$, u_0 is the actual control single.

A. Observer Design

Considering the System (1), we introduce the following auxiliary function

$$F_i(\underline{x}_i, x_{i+1}) = f_i(\underline{x}_i, x_{i+1}) - x_{i+1}, \tag{7}$$

where $i = 1, \cdots, n-1$.

Assumption 2: For $\forall M_1 \in \mathbb{R}^j$ and $\forall M_2 \in \mathbb{R}^j$, there exit known constants m_i satisfying that

$$|F_i(M_1) - F_i(M_2)| \leq m_i ||M_1 - M_2||, \tag{8}$$

where $i = 1, \cdots, n$, and j is appropriate dimension of the vector.

It follows, from (7), that (1) can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2 + F_1\left(x_1, x_2\right), \\ \dot{x}_2 = x_3 + F_2\left(\underline{x}_2, x_3\right), \\ \qquad \vdots \\ \dot{x}_{n-1} = x_n + F_{n-1}\left(\underline{x}_{n-1}, x_n\right), \\ \dot{x}_n = \varpi^{\mathrm{T}}\underline{u} + F_n(\underline{x}_n), \\ y = x_1. \end{cases} \tag{9}$$

Define $\hat{x}_i$ as the estimation value of x_i, and the filtered signals

$$\hat{x}_{i,f} = B_{\mathrm{L}}(s)\hat{x}_i, \tag{10}$$

where $B_{\mathrm{L}}(s)$ is a Butterwoth low-pass filter (BLF), and corresponding parameters of the filter whose cutoff frequency $w_c = 1\mathrm{rad} \cdot \mathrm{s}^{-1}$ with different orders are listed in [31].

Assumption 3: There exits positive a constant $b_{i,0}$, such that $|\hat{x}_i - \hat{x}_{i,f}| \leq b_{i,0}$, $i = 1, \cdots, n$.

Define $\Delta F_i = F_i(\underline{x}_i, x_{i+1}) - F_i(\underline{\hat{x}}_i, \hat{x}_{i+1,f})$, $\Delta F_n = F_n(\underline{x}_{n-1}, x_n) - F_n(\underline{\hat{x}}_{n-1}, \hat{x}_n)$, and we can obtain that

$$\begin{cases} \dot{x}_1 = x_2 + F_1\left(\underline{\hat{x}}_1, \hat{x}_{2,f}\right) + \Delta F_1, \\ \dot{x}_2 = x_3 + F_2\left(\underline{\hat{x}}_2, \hat{x}_{3,f}\right) + \Delta F_2, \\ \qquad \vdots \\ \dot{x}_{n-1} = x_n + F_{n-1}\left(\underline{\hat{x}}_{n-1}, \hat{x}_{n,f}\right) + \Delta F_{n-1}, \\ \dot{x}_n = \varpi^{\mathrm{T}}\underline{u} + F_n\left(\underline{\hat{x}}_{n-1}, \hat{x}_n\right) + \Delta F_n, \\ y = x_1. \end{cases} \tag{11}$$

Further, (11) can be rewritten in the vector form

$$\begin{cases} \underline{\dot{x}} = A\underline{x} + Ky + \sum_{i=1}^{n-1} \Big[B_i F_i(\underline{\hat{x}}_i, \hat{x}_{i+1,f})\Big] \\ \qquad + B_n \Big[F_n\big(\underline{\hat{x}}_{n-1}, \hat{x}_n\big) + \varpi^{\mathrm{T}} \underline{u}\Big] + \Delta F, \\ y = C\underline{x}, \end{cases} \tag{12}$$

where $A = \begin{bmatrix} -k_1 & 1 & 0 & \cdots & 0 & 0 \\ -k_2 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ -k_{n-1} & 0 & 0 & \cdots & 0 & 1 \\ -k_n & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}_{n\times n}$, $K = [k_1, \cdots, k_n]^{\mathrm{T}}$, $B = \Big[\overbrace{0, \cdots, 0}^{i-1}, 1, \underbrace{0, \cdots, 0}_{n-i}\Big]^{\mathrm{T}}$, $C = \Big[\overbrace{0, \cdots, 0}^{n-1}, 1\Big]^{\mathrm{T}}$, $\Delta F = [\Delta F_1, \cdots, \Delta F_n]^{\mathrm{T}}$, A is a strict Hurwitz matrix satisfying that there exists a positive matrix $P = P^{\mathrm{T}}$, for a given positive symmetric matrix Q, such that

$$A^{\mathrm{T}} P + PA = -Q. \tag{13}$$

According to the approximation theory, the unknown functions $F_i(\underline{\hat{x}}_i, \hat{x}_{i,f})$ and $F_n(\underline{\hat{x}}_{n-1}, \hat{x}_n)$ can be written as

$$F_i(\underline{\hat{x}}_i, \hat{x}_{i+1,f}) = W_i^* G_i(\underline{\hat{x}}_i, \hat{x}_{i+1,f}) + \varepsilon_i, i = 1, \cdots, n-1, \tag{14}$$

and

$$F_n(\underline{\hat{x}}_{n-1}, \hat{x}_n) = W_n^* G_n(\underline{\hat{x}}_{n-1}, \hat{x}_n) + \varepsilon_n, \tag{15}$$

respectively, where W_i^*, W_n^*, $G_i(\cdot)$ and $G_n(\cdot)$ are introduced in Section II, ε_i and ε_n are the approximation errors satisfying $|\varepsilon_i| \le \varepsilon_i^*$, $|\varepsilon_n| \le \varepsilon_n^*$, ε_i^* and ε_n^* are constant.

We denote that $Z_i = \big[\underline{\hat{x}}_i, \hat{x}_{i,f}\big]^{\mathrm{T}}$ and $Z_n = \big[\underline{\hat{x}}_{n-1}, \hat{x}_n\big]^{\mathrm{T}}$, and a state observer is proposed for the System (1)

$$\begin{cases} \underline{\dot{\hat{x}}} = A\underline{\hat{x}} + Ky + \sum_{i=1}^{n-1} B_i \hat{W}_i^{\mathrm{T}} G_i(Z_i) + B_n \hat{W}_n^{\mathrm{T}} G_n(Z_n) \\ \qquad + B_n \varpi^{\mathrm{T}} \underline{u}, \\ \hat{y} = C\underline{\hat{x}}, \end{cases} \tag{16}$$

where $\hat{W}_i$ and $\hat{W}_n$ are the estimation values of W_i^* and W_n^*, respectively, and their adaptive update laws will be given later.

Define the observer error vector $\underline{\widetilde{x}} = \underline{x} - \underline{\hat{x}}$, and the derivative can be expressed as

$$\begin{aligned} \underline{\dot{\widetilde{x}}} &= \underline{\dot{x}} - \underline{\dot{\hat{x}}} \\ &= A\underline{\widetilde{x}} + \sum_{i=1}^{n-1} \Big[B_i W_i^{*\mathrm{T}} G_i(Z_i) - B_i \hat{W}_i^{\mathrm{T}} G_i(Z_i)\Big] \\ &\quad + B_n W_n^{*\mathrm{T}} G_n(Z_n) - B_n \hat{W}_n^{\mathrm{T}} G_n(Z_n) + \Delta F + \varepsilon \\ &= A\underline{\widetilde{x}} + \Delta F + \delta + \varepsilon, \end{aligned} \tag{17}$$

where $\delta = [\delta_1, \cdots, \delta_n]^{\mathrm{T}}$, $\varepsilon = [\varepsilon_1, \cdots, \varepsilon_n]^{\mathrm{T}}$, $\delta_p = \widetilde{W}_p^{\mathrm{T}} G_p(Z_p)$, $\delta_n = \widetilde{W}_n^{\mathrm{T}} G_n(Z_n)$, $\widetilde{W}_p = W_p^* - \hat{W}_p$, $p = 1, \cdots, n-1$, $\widetilde{W}_n = W_n^* - \hat{W}_n$.

B. Control Law Design

In this part, we will design an adaptive output feedback fault-tolerant control scheme for the System (1).

Define

$$s_1 = x_1 - y_{\mathrm{d}}, \tag{18}$$

$$s_i = \hat{x}_i - z_i, \tag{19}$$

where z_i is obtained from a filtered signal of virtual control, $i = 1, \cdots, n$.

Step 1: Differentiating s_1 and considering the System (1), we obtain that

$$\begin{aligned} \dot{s}_1 &= \dot{x}_1 - \dot{y}_{\mathrm{d}} \\ &= x_2 + F_1\left(\underline{\hat{x}}_{i,1}, \hat{x}_{i,2,f}\right) + \Delta F_1 - \dot{y}_{i,\mathrm{d}} \\ &= \hat{x}_2 + \widetilde{x}_2 + W_1^{*\mathrm{T}} G_1(Z_1) + \varepsilon_1 + \Delta F_1 - \dot{y}_{\mathrm{d}}. \end{aligned} \tag{20}$$

Choose the virtual control law α_2

$$\alpha_2 = -k_1 s_1 - \hat{W}_1^{\mathrm{T}} G_1(Z_1) + \dot{y}_{\mathrm{d}}, \tag{21}$$

and the update law for $\hat{W}_1$

$$\dot{\hat{W}}_1 = \varGamma_1 \Big[G_1(Z_1) s_1 - \sigma_1 \hat{W}_1\Big], \tag{22}$$

where $k_1 > 0$, $\varGamma_1 = \varGamma_1^{\mathrm{T}} > 0$, $\sigma_1 > 0$.

DSC technique in [22] is introduced here, and let α_2 pass through a first-order filter to obtain z_2

$$\varsigma_2 \dot{z}_2 + z_2 = \alpha_{i,2}, \ z_2(0) = \alpha_2(0), \tag{23}$$

where $\varsigma_2 > 0$.

Step 2: Let $s_2 = \hat{x}_2 - z_2$. According to the System (1) and the observer (16), we have

$$\begin{aligned} \dot{s}_2 &= \dot{\hat{x}}_2 - \dot{z}_2 \\ &= -k_2 \hat{x}_1 + \hat{x}_3 + k_2 y + W_2^{*\mathrm{T}} G_2(Z_2) + \varepsilon_2 - \dot{z}_2. \end{aligned} \tag{24}$$

We construct the virtual control law

$$\alpha_3 = -k_2 s_2 + k_2 \hat{x}_1 - k_2 y - \hat{W}_2^{\mathrm{T}} G_2(Z_2) + \dot{z}_2 \tag{25}$$

with the adaptive tuning rule

$$\dot{\hat{W}}_2 = \varGamma_2 \Big[G_2(Z_2) s_2 - \sigma_2 \hat{W}_2\Big], \tag{26}$$

where $k_2 > 0, \varGamma_2 = \varGamma_2^{\mathrm{T}} > 0, \sigma_2 > 0$.

Similarly, define a first-order filter

$$\varsigma_3 \dot{z}_3 + z_3 = \alpha_3, \ z_3(0) = \alpha_3(0), \tag{27}$$

where $\varsigma_3 > 0$.

Step i ($3 \le i \le n-1$): Similarly to the previous steps,

$$\begin{aligned} \dot{s}_i &= \dot{\hat{x}}_i - \dot{z}_i \\ &= -k_i \hat{x}_1 + \hat{x}_{i+1} + k_i y + W_i^{*\mathrm{T}} G_i(Z_i) + \varepsilon_i - \dot{z}_i. \end{aligned} \tag{28}$$

Accordingly, we choose

$$\alpha_{i+1} = -k_i s_i + k_i \hat{x}_1 - k_i y - \hat{W}_i^{\mathrm{T}} G_i(Z_i) + \dot{z}_i \tag{29}$$

with

$$\dot{\hat{W}}_i = \varGamma_i \Big[G_i(Z_i) s_i - \sigma_i \hat{W}_i\Big], \tag{30}$$

where $k_i > 0, \mathit{\Gamma}_i = \mathit{\Gamma}_i^{\mathrm{T}} > 0, \sigma_i > 0$.

Similarly, a first-order filter

$$\varsigma_{i+1}\dot{z}_{i+1} + z_{i+1} = \alpha_{i+1},\ z_{i+1}(0) = \alpha_{i+1}(0), \tag{31}$$

is introduced, where $\varsigma_{i+1} > 0$.

Step n: From (19), we have

$$\begin{aligned}\dot{s}_n &= \dot{\hat{x}}_n - \dot{z}_n \\ &= -k_n\hat{x}_1 + \varpi^{\mathrm{T}}\underline{u} + k_n y + W_n^{*\mathrm{T}}G_n(Z_n) + \varepsilon_n - \dot{z}_n. \end{aligned}\tag{32}$$

Then, the control law is designed as

$$\begin{aligned}u_0 = \frac{1}{\hbar_0}\Big[&- k_n s_n - \sum_{j=j_1\cdots j_q}\varpi_j\bar{u}_j + k_n\hat{x}_1 - k_n y \\ &- \hat{W}_n^{\mathrm{T}}G_n(Z_n) + \dot{z}_n\Big]\end{aligned}\tag{33}$$

with the adaptive law

$$\dot{\hat{W}}_n = \mathit{\Gamma}_n\Big[G_n(Z_n)s_n - \sigma_n\hat{W}_n\Big], \tag{34}$$

where $k_n > 0$, $\hbar_0 = \sum_{j\neq j_1\cdots j_q}\kappa_j b_j$, $\mathit{\Gamma}_n = \mathit{\Gamma}_n^{\mathrm{T}} > 0$, $\sigma_n > 0$.

C. Stability Analysis

In this section, the stability of the closed-loop system will be analysed.

Define $e_{i+1} = z_{i+1} - \alpha_{i+1}$, and according to (23), $\dot{z}_{i+1}$ can be expressed as

$$\begin{aligned}\dot{z}_{i+1} &= \frac{\alpha_{i+1} - z_{i+1}}{\varsigma_{i+1}} \\ &= -\frac{e_{i+1}}{\varsigma_{i+1}},\end{aligned}\tag{35}$$

where $i = 1, \cdots, n-1$.

Combing (21) and (35), we can obtain that

$$\begin{aligned}\dot{e}_2 &= \dot{z}_2 - \dot{\alpha}_2 \\ &= -\frac{e_2}{\varsigma_2} + k_1\dot{s}_1 + \dot{\hat{W}}_1^{\mathrm{T}}G_1(Z_1) + \frac{\hat{W}_1^{\mathrm{T}}\partial G_1(Z_1)}{\partial Z_1}\dot{Z}_1 + \ddot{y}_{\mathrm{d}} \\ &= -\frac{e_2}{\varsigma_2} + B_2\Big(s_1, s_2, e_2, Z_1, \hat{W}_1, y_{\mathrm{d}}, \dot{y}_{\mathrm{d}}, \ddot{y}_{\mathrm{d}}\Big),\end{aligned}\tag{36}$$

where $B_2(\cdot)$ is a continuous function.

Similarly, for $j = 2, \cdots, n-1$,

$$\begin{aligned}\dot{e}_{i+1} &= \dot{z}_{i+1} - \dot{\alpha}_{i+1} \\ &= -\frac{e_{i+1}}{\varsigma_{i+1}} + k_i\dot{s}_i - k_i\dot{\hat{x}}_1 + k_i\dot{y} + \dot{\hat{W}}_i^{\mathrm{T}}G_i(Z_i) \\ &\quad + \frac{\hat{W}_i^{\mathrm{T}}\partial G_i(Z_i)}{\partial Z_i}\dot{Z}_i - \frac{\dot{e}_i}{\varsigma_i} \\ &= -\frac{e_{i+1}}{\varsigma_{i+1}} + B_{i+1}\Big(s_1, \cdots, s_{i+1}, e_2, \cdots, e_{i+1}, Z_i, \\ &\qquad \hat{W}_1, \cdots, \hat{W}_i, y_{\mathrm{d}}, \dot{y}_{\mathrm{d}}, \ddot{y}_{\mathrm{d}}\Big),\end{aligned}\tag{37}$$

where $B_{i+1}(\cdot)$ is a continuous function.

Choose $V_o = \underline{\widetilde{x}}^{\mathrm{T}}P\underline{\widetilde{x}}$, and the derivative of V_o is

$$\begin{aligned}\dot{V}_o &= \dot{\underline{\widetilde{x}}}^{\mathrm{T}}P\underline{\widetilde{x}} + \underline{\widetilde{x}}^{\mathrm{T}}P\dot{\underline{\widetilde{x}}} \\ &= -\underline{\widetilde{x}}^{\mathrm{T}}Q\underline{\widetilde{x}} + 2\underline{\widetilde{x}}^{\mathrm{T}}P\delta + 2\underline{\widetilde{x}}^{\mathrm{T}}P\Delta F + 2\underline{\widetilde{x}}^{\mathrm{T}}P\varepsilon.\end{aligned}\tag{38}$$

Using Assumption 2, Assumption 3 and the facts that

$$\begin{aligned}\Delta F_i \le\ & |F_i(\underline{x}_i, x_{i+1}) - F_i(\underline{\hat{x}}_i, \hat{x}_{i+1})| \\ & + |F_i(\underline{\hat{x}}_i, \hat{x}_{i+1}) - F_i(\underline{\hat{x}}_i, \hat{x}_{i+1,f})| \\ \le\ & m_i||\underline{\widetilde{x}}|| + b'_{i+1,0}, \qquad i = 1, \cdots, n-1,\end{aligned}\tag{39}$$

$$\begin{aligned}\Delta F_n \le\ & |F_n(\underline{x}_{n-1}, x_n) - F_n(\underline{\hat{x}}_{n-1}, \hat{x}_n)| \\ \le\ & m_n||\underline{\widetilde{x}}|| + b'_{n+1,0},\end{aligned}\tag{40}$$

$$\delta_i^2 \le\ g_i^{*2}\widetilde{W}_i^{\mathrm{T}}\widetilde{W}_i, \tag{41}$$

the following inequalities hold

$$\begin{aligned}2\underline{\widetilde{x}}^{\mathrm{T}}P\delta \le\ & \underline{\widetilde{x}}^{\mathrm{T}}\underline{\widetilde{x}} + ||P||^2\Big(|\delta_1|^2 + \cdots + |\delta_n|^2\Big) \\ \le\ & \underline{\widetilde{x}}^{\mathrm{T}}\underline{\widetilde{x}} + ||P||^2\sum_{i=1}^{n}g_i^{*2}\widetilde{W}_i^{\mathrm{T}}\widetilde{W}_i,\end{aligned}\tag{42}$$

$$\begin{aligned}2\underline{\widetilde{x}}^{\mathrm{T}}P\Delta F \le\ & \underline{\widetilde{x}}^{\mathrm{T}}\underline{\widetilde{x}} + ||P||^2\Big(|\Delta F_1|^2 + \cdots + |\Delta F_n|^2\Big) \\ \le\ & \underline{\widetilde{x}}^{\mathrm{T}}\underline{\widetilde{x}} + ||P||^2\sum_{i=1}^{n}\Big(2m_i^2||\underline{\widetilde{x}}||^2 + 2b_{i+1,0}^{\prime 2}\Big) \\ \le\ & c_0||\underline{\widetilde{x}}||^2 + q_0,\end{aligned}\tag{43}$$

$$2\underline{\widetilde{x}}^{\mathrm{T}}P\varepsilon \le\ \underline{\widetilde{x}}^{\mathrm{T}}\underline{\widetilde{x}} + ||P||^2||\varepsilon^*||^2, \tag{44}$$

where $g_i^* > 0$ is the upper bound of $||G_i(Z_i)||$ [30], $b'_{i+1,0} = m_i b_{i+1,0}$, $b_{n+1,0} = 0$ is for notation convenience, $c_0 = 1 + 2||P||^2\sum_{i=1}^{n}m_i^2$, $q_0 = 2||P||^2\sum_{i=1}^{n}b_{i+1,0}^{\prime 2}$,

In view of (38)–(44), we have

$$\begin{aligned}\dot{V}_o \le\ & -\underline{\widetilde{x}}^{\mathrm{T}}(Q - 2I - c_0 I)\underline{\widetilde{x}} + ||P||^2\sum_{i=1}^{n}g_i^{*2}\widetilde{W}_i^{\mathrm{T}}\widetilde{W}_i \\ & + q_0 + ||P||^2||\varepsilon^*||^2.\end{aligned}\tag{45}$$

The main result of this paper is summarized in the following theorem.

Theorem 1: Considering the System (1) in the presence of actuator faults (2) and (3), we design the observer (16), the controller (33) and adaptive laws (22)(26)(30)(34) under the condition that Assumption 1–Assumption 3 hold. For bounded initial conditions, all signals of the closed-loop system are uniformly ultimately bounded and the tracking error remains in a small neighborhood around the origin with suitable choice of design parameters.

Proof 1: Choose the following Lyapunov function candidate

$$V = V_o + \frac{1}{2}\sum_{i=1}^{n}s_i^2 + \frac{1}{2}\sum_{i=1}^{n}\widetilde{W}_i^{\mathrm{T}}\mathit{\Gamma}_i^{-1}\widetilde{W}_i + \frac{1}{2}\sum_{i=1}^{n-1}e_{i+1}^2, \tag{46}$$

and its differentive yields

$$\dot{V} = \dot{V}_o + \sum_{i=1}^{n}s_i\dot{s}_i + \sum_{i=1}^{n}\widetilde{W}_i^{\mathrm{T}}\mathit{\Gamma}_i^{-1}\dot{\widetilde{W}}_i + \sum_{i=1}^{n-1}e_{i+1}\dot{e}_{i+1}. \tag{47}$$

From Assumption 1, Assumption 2, and Young's inequality, we have the following inequalities

$$|s_1\Delta F_1(\cdot)|\leq \frac{s_1^2}{4}+2\Big(m_1^2||\underline{\widetilde{x}}||^2+b_{2,0}^{'2}\Big), \tag{48}$$

$$|s_1\widetilde{x}_2|\leq \frac{s_1^2}{4}+\underline{\widetilde{x}}^{\mathrm{T}}\underline{\widetilde{x}}, \tag{49}$$

$$\sigma_i\widetilde{W}_i^{\mathrm{T}}\hat{W}_i\leq -\frac{\sigma_i}{2}\widetilde{W}_i^{\mathrm{T}}\widetilde{W}_i+\frac{\sigma_i}{2}W_i^{*\mathrm{T}}W_i^*, \tag{50}$$

$$e_{i+1}B_{i+1}(\cdot)\leq \frac{M_{i+1}^2}{2\beta_{i+1}}+\frac{\beta_{i+1}e_{i+1}^2}{2}. \tag{51}$$

(47) can be rewritten as

$$\begin{aligned}\dot{V}\leq & -\underline{\widetilde{x}}^{\mathrm{T}}\Big(Q-3I-c_0I-2m_1^2I\Big)\underline{\widetilde{x}}+\Big(-k_1+\frac{1}{2}\Big)s_1^2\\ &+\sum_{i=1}^{n-1}\Big(s_is_{i+1}+s_ie_{i+1}-k_is_i^2\Big)+\sum_{i=1}^{n}s_i\varepsilon_i\\ &-k_ns_n^2+\sum_{i=1}^{n}\Big[\big(||P||^2g_i^{*2}-\frac{\sigma_i}{2}\big)\widetilde{W}_i^{\mathrm{T}}\widetilde{W}_i\Big]\\ &+\sum_{i=1}^{n-1}\Big(-\frac{e_{i+1}^2}{\varsigma_{i+1}}+\frac{\beta_{i+1}e_{i+1}^2}{2}\Big)+q_0+2b_{2,0}^{'2}\\ &+||P||^2||\varepsilon^*||^2+\sum_{i=1}^{n}\frac{\sigma_i}{2}W_i^{*\mathrm{T}}W_i^*+\sum_{i=1}^{n-1}\frac{M_{i+1}^2}{2\beta_{i+1}}, \end{aligned}\tag{52}$$

where β_{i+1} is positive constant and M_{i+1} is the maximum value of the the continuous function $B_{i+1}(\cdot)$.

Since

$$s_is_{i+1}\leq s_i+\frac{1}{4}s_{i+1}^2, \tag{53}$$

$$s_ie_{i+1}\leq s_i+\frac{1}{4}e_{i+1}^2, \tag{54}$$

$$s_i\varepsilon_i\leq s_i+\frac{1}{4}\varepsilon_i^2\leq s_i+\frac{1}{4}\varepsilon_i^{*2}, \tag{55}$$

and denoting that $\frac{1}{\varsigma_{i+1}}=\frac{\beta_{i+1}}{2}+\frac{1}{4}+\varsigma_{i+1}^*$, $\varsigma_{i+1}^*>0$, the inequality (52) can be further written as

$$\begin{aligned}\dot{V}\leq & -\underline{\widetilde{x}}^{\mathrm{T}}\Big(Q-2I-c_0I-2m_1^2I\Big)\underline{\widetilde{x}}-\Big(k_1-\frac{7}{2}\Big)s_1^2\\ &-\sum_{i=2}^{n-1}\Big(k_i-\frac{13}{4}\Big)s_i^2-\Big(k_n-\frac{5}{4}\Big)s_n^2-\sum_{i=1}^{n-1}\varsigma_{i+1}^*e_{i+1}^2\\ &+\sum_{i=1}^{n}\Big(||P||^2g_i^{*2}-\frac{\sigma_i}{2}\Big)\widetilde{W}_i^{\mathrm{T}}\widetilde{W}_i+q_0+2b_{2,0}^{'2}+\sum_{i=1}^{n}\frac{\varepsilon_i^{*2}}{4}\\ &+||P||^2||\varepsilon^*||^2+\sum_{i=1}^{n}\frac{\sigma_i}{2}W_i^{*\mathrm{T}}W_i^*+\sum_{i=1}^{n-1}\frac{M_{i+1}^2}{2\beta_{i+1}}\\ \leq & -\chi V+C, \end{aligned}\tag{56}$$

where $\chi=\min\Big\{\lambda_{\min}\big[(Q-3I-c_0I-2m_1^2I)P^{-1}\big],2(k_1-7/2),2\min(k_i-13/4,\cdots,k_{n-1}-13/4),2(k_n-5/4),2\min(\varsigma_2^*,\cdots,\varsigma_n^*),2\min\big[(\frac{\sigma_1}{2}-||P||^2g_1^{*2})/\lambda_{\max}(\Gamma_1^{-1}),\cdots,(\frac{\sigma_n}{2}-||P||^2g_n^{*2})/\lambda_{\max}(\Gamma_n^{-1})\big]\Big\}$, $C=q_0+2b_{2,0}^{'2}+\sum_{i=1}^{n}\frac{\varepsilon_i^{*2}}{4}+||P||^2||\varepsilon^*||^2+\sum_{i=1}^{n}\frac{\sigma_i}{2}W_i^{*\mathrm{T}}W_i^*+\sum_{i=1}^{n-1}\frac{M_{i+1}^2}{2\beta_{i+1}}$.

From (56), one has

$$0\leq V(t)\leq \frac{C}{\chi}+\Big[V(0)-\frac{C}{\chi}\Big]\mathrm{e}^{-\chi t}. \tag{57}$$

It indicates that all signals of the closed-loop system are uniformly ultimately bounded. And the analysis about convergence of the tracking error is the same as [19]. This is omitted for the sake of space limitation.

IV. Conclusion

In this paper, the adaptive fault-tolerant control strategy has been presented for a class of uncertain nonlinear systems suffered from actuator failures. The main advantage is that it is assumed that not all inner states are available and the control strategy is proposed combing a NNs-observer with the bacstepping approach and DSC technique. The stability of the whole system and the convergence of the tracking error are shown effectively via theoretical proof.

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